Name: ______________________________________ Date: ___________________

A2CC Shifting Functions

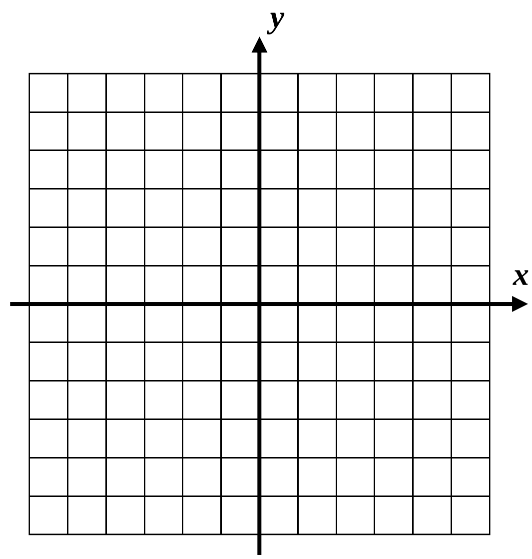
The basic geometric transformations of translating (shifting), reflecting, rotating, and dilating can all be done to the graphs of functions by algebraically manipulating them. Rotating functions is out of the scope of our work in this course, but we will investigate all of the others, all of which you originally saw in Common Core Algebra I. In this lesson, we will concentrate on shifting functions.

***Exercise* #1:** Consider the functions $y=|x|$, $y=|x-3|+2$, and $y=|x+1|-4$.

(a) Without the use of your calculator, graph $y=|x|$ on the axes provided. Label its equation.

(b) Using your calculator to generate a table of values, graph the other two absolute value functions above and label each with its equation.

(c) How would the graph of $y=|x|$ be shifted in order to produce the graph of $y=|x-6|-8$?

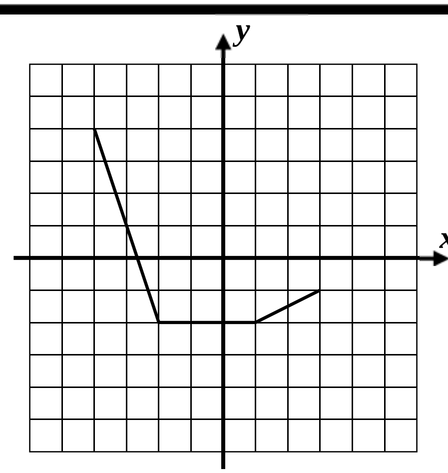


Although we just used the absolute value function, this exercise illustrates the fundamental manner in which vertical shifts and horizontal shifts occur in functions.

VERTICAL AND HORIZONTAL SHIFTING

1. **Vertical Shifting**: The function $f(x)+k$ shifts the function up by $|k|$ units for $k>0$ and down $|k|$ units for $k<0$.
2. **Horizontal Shifting**: The function $f(x+k)$ shifts the function left $|k|$ units for $k>0$ and right $|k|$ units for $k<0$.

***Exercise* #2:** The function $f(x)$ is shown on the grid below. A second function, *g*, is defined by $g(x)=f(x-3)+1$.



(a) What is the value of $g(0)$? Show how you arrived at your answer.

(b) Identify how the graph of *f* has been transformed to produce the graph of *g* and sketch it on the grid.

We can use these shifting patterns in a variety of ways because they apply to all types of functions.

***Exercise* #3:** A function, $f(x)$, has a domain of $-3 \le x \le 10$ and a range of $y \le 22$. What are the domain and range of the function $f(x+7)+10$? Explain how you arrived at your answers.

Recognizing shifts of other, simpler functions can help us identify prominent characteristics and compare them. The location of turning points is especially helpful.

***Exercise* #4:** Given the quadratic function $f(x)=(x-4)^2-5$ answer the following questions.

(a) How has the simple quadratic $y=x^2$ been shifted to produce the graph of $f(x)$?

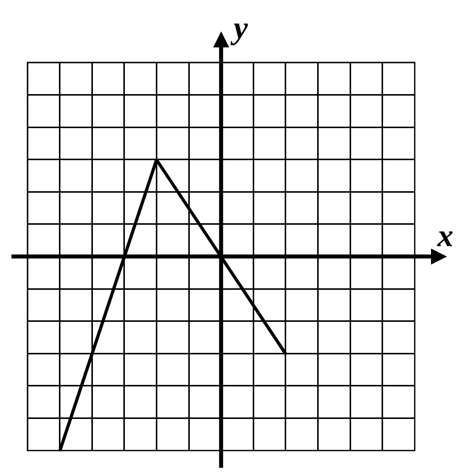
(b) Given that $y=x^2$ has a turning point at the origin, $(0,0)$, where must the turning point of *f* lie?

(c) Sketch *f* below and give the domain interval over which *f* is increasing.

(d) Which has a lower minimum value, the function *f* or the function $g(x)=|x-6|-10$? Explain your choice.

One of the hardest things for students to grasp is the horizontal shift, which appears to work opposite of what we would expect. Let's take a look at a shift that is purely horizontal.

***Exercise* #5:** The graph of $f(x)$ is shown below. The function $g(x)$ is defined by $g(x)=f(x-2)$.



(a) Show that $x=-1$ and $x=2$ are zeroes of the function *g*.

(b) Evaluate each of the following using the definition of *g* and then create a plot of *g* on the same set axes.

$g(-3)=$ $\qquad$ $g(0)=$ $\qquad$ $g(4)=$

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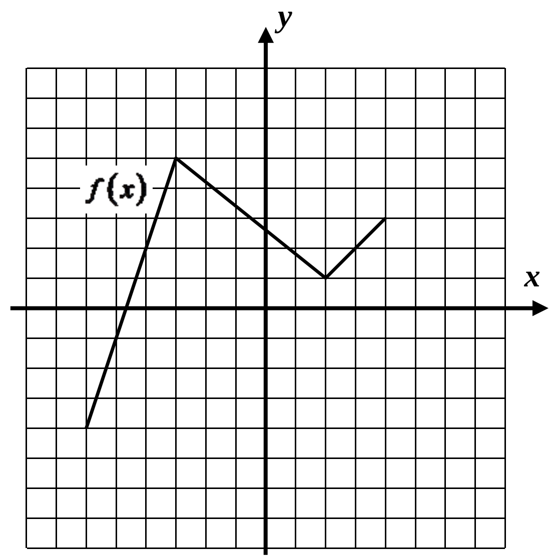
A2CC Shifting Functions

SHIFTING FUNCTIONS
HOMEWORK

1. Given the function $f(x)$ shown graphed on the grid, create a graph for each of the following functions and label on the grid.



(a) $g(x) = f(x) + 2$

(b) $h(x) = f(x - 3)$

(c) $k(x) = f(x + 1) - 4$

2. If the quadratic function $f(x)$ has a turning point at $(-3, 7)$ then where does the quadratic function g defined by $g(x) = f(x+4)+5$ have a turning point?

(1) $(-7, 12)$ (3) $(-4, 5)$

(2) $(1, 12)$ (4) $(4, 5)$

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3. Over which of the following intervals would the function $h(x) = |x-2|+6$ be decreasing only? Sketch a graph of the function if needed.

(1) $x > 2$ (3) $x < 6$

(2) $x < 2$ (4) $x > 6$

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4. If the domain of $f(x)$ is $-3 \le x \le 9$ and the range of $f(x)$ is $2 \le y \le 15$, then which of the following statements is correct about the domain and range of $g(x) = f(x-2) - 8$?

(1) Its domain is $-1 \le x \le 11$ and its range is $10 \le y \le 23$.

(2) Its domain is $-5 \le x \le 7$ and its range is $-6 \le y \le 7$.

(3) Its domain is $-1 \le x \le 11$ and its range is $-6 \le y \le 7$.

(4) Its domain is $-5 \le x \le 7$ and its range is $10 \le y \le 23$.

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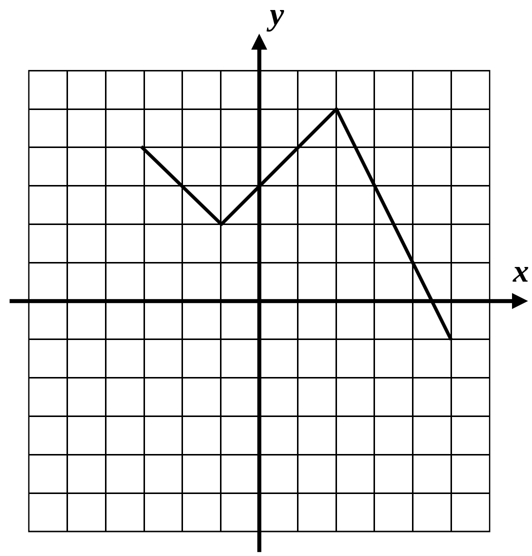
5. The graph of the function $f(x)$ is shown on the grid below. The function g is defined by the formula $g(x) = f(x+3) - 1$.



(a) Graph and label g on the axes along with f.

(b) What is the smallest solution to the equation $f(x) = g(x)$?

(c) If $h(x) = g(x) - 3$, explain why the equation $h(x) = f(x)$ has no solutions.

APPLICATIONS

6. A projectile has a height given by the function $h(t) = -4.9(t-4)^2 + 153$, where times, t, is in seconds and the height, h, is in meters. What is the maximum height of the function and at what time does it reach that height?

REASONING

7. Given the linear equations $f(x) = 2x$ and $g(x) = 2x - 2$ answer the following.

(a) Show that the function f passes through the origin.

(b) How has the function f been shifted to produce the function g?

(c) Write the function g in factored form.

(d) Based on (c), how has the function f been shifted to produce the function g?

(e) How would $f(x)$ need to be shifted to produce $h(x) = 2(x+5) - 7$? Given that f must contain the point $(0, 0)$, what point must $h(x)$ contain based on the shifting?